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CUEING PERFORMANCE ESTIMATION USING SPACE BASED OBSERVATIONS DURING BOOST PHASE

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ABSTRACT

This paper addresses error statistics for estimates of ballistic missile trajectory parameters that are computed from observations by space-based infrared (IR) sensors during the boost phase. These error statistics are useful for system level planning and performance estimation of such quantities as cueing accuracy vs time, sensitivity to satellite parameters, or impact point predictions. These statistics assume correct ballistic missile typing, which provides the basis for most of the inputs on which we have based our calculations. We simplify the actual ballistic missile motions to derive error estimates, but do assume both random and bias line-of-sight (LOS) angular errors from the satellites. The results of these error analyses compare favorably with much more detailed 6 degree of freedom simulation analyses and the results are easily embedded in spread sheets. Error statistics are given for the estimates of launch points, azimuth, cross range and down range impact points, and cueing area growth rate.

INTRODUCTION

An earlier version of this paper with the same title has been published in The Proceedings of the 1996 Summer Computer Simulation Conference, which was held in Portland, Oregon on July 21-26,1996. This version contains some corrections and extensions.

The purpose of this paper is to provide rules of thumb to estimate the performance of space-based IR surveillance systems observing ballistic missiles during boost phase. It is intended to foster insight and understanding for top-level system design and performance analyses. It facilitates a spread-sheet approach to enable quick and easy variation of input parameters. Given information about the locations of viewing satellites and their ballistic missile target and the random and bias errors of the satellites, ballistic missile launch and trajectory parameter accuracies and their error statistics can be estimated. These parameters include the launch and impact point locations, the azimuth and elevation angles of the ballistic missile flight path, and the ballistic state vector at burnout, which allows predictions of future positions,. The derived error statistics of these parameters are provided as standard deviations or percentile estimates of the errors of each parameter, thus enabling bounding, for example, of the system-level estimates of error volumes to be searched, the attack area uncertainty for retaliation against the transporter erector launcher (TEL), or the potential area threatened by a ballistic missile attack that might require personnel to take shelter.

The nominal error calculations are made assuming stereo viewing, with modifications to account for Nsat (i.e. number of satellites) viewing. This paper provides approximations and supporting rationale for the error statistics of the parameters being estimated but **does not attempt to actually estimate the parameters**. The results can be viewed as analogous to a Cramer-Rao lower bound on the estimate accuracy or the result of a detailed covariance analysis. It should be expected that real-world results would generally be more pessimistic because of violations of the simplifying assumptions.

The actual tracking processes used to estimate the parameters are not modeled, but assumptions are made about the general tracking approaches used. The equations are based on empirical fits to Monte-Carlo data, linear estimates of processes known to be much more complex, and simplified statistical assumptions (usually normal or uniform). However, the results appear to be reasonable (i.e., within about 10 percent) when compared to more sophisticated analyses and actual performance in some cases.

ASSUMPTIONS

Some of the simplifying assumptions used in this analysis are as follows

1. This work was performed at POET and supported by the Ballistic Missile Defense Organization under IDA Contract Number DASW-01094-c-0054 and MITRE Contract Number DAAB-07-96-C-E601

- Each satellite has random and bias angular observation errors that are normally distributed and independent from each other with zero mean and known standard deviations. The random errors are σazr and σelr for random errors along the LOS in azimuth and elevation, respectively. They will produce different errors for each observation. The bias errors, σazb and σelb, are similarly zero mean and known standard deviation, but they are constant for each series of observations for a given satellite. In general, the standard deviations of the azimuth and elevation errors for a given satellite will be considered to be the same.

- The target is located in the mutual stereo viewing area of the satellite pairs, and individual observations are made at random times by each satellite, with a constant revisit time, Tr, between observations by a given satellite. The revisit time is the most important single parameter effecting errors in estimating future positions because it determines the range of uncertainty in the burnout time for the ballistic missile, Tbo, which is estimated as the midpoint of the time between the last observation that observes the ballistic missile and the first observation opportunity that does not observe the ballistic missile. An additional assumption is that the observing sensitivity is adequate to detect the ballistic missile at all times up to burnout. Although the observations will be considered to occur at random times for estimation of burnout time, for simplicity in the derivation of the other error statistics it will be assumed that a stereo observation consists of two simultaneous LOS observations.

- Enough observable burn time is present after cloud break for at least one additional observation by each satellite of a stereo pair after an initial observation by that satellite. The number of stereo observation intervals, n, must be at least one, or the motion of the ballistic missile cannot be estimated.

- The satellite and the ballistic missile target positions are assumed, establishing the fundamental viewing geometry. The type of target is known, its nominal flight path template (i.e., down-range distance and altitude vs. time) is known, and the ballistic missile exhibits no anomalous behavior. No sensor-sensor correlation errors exist between the satellites.

Note: These assumptions are heroic. If they are violated, the validity of the following results can be negated.

INPUT DATA

To illustrate the process of estimating trajectory errors in a general way and to avoid any hint of the use of classified data, some notional Theater Ballistic Missile (TBM) performance data (that empirically approximates a minimum energy TBM trajectory) and satellite LOS errors are offered to provide a frame of reference (see Table 1).

Table 1. Input Data

Satellite Data
LOS random error
σazr = σelr = 15 microradians
LOS bias error
σazb = σelb = 50 microradians
Satellite positions are geosynchronous at
30° W / 30° E longitude
Revisit time
Tr = 10 seconds
TBM Target Data
TBM Target launch position
35° N latitude, 0° E longitude
Cloud break time
cb = 30 seconds
TBM Flight Data
(rng = range of TBM in km, Re = earth radius in km)
Burn time
bt = 9.4 * $rng^{1/3}$ seconds
Burn distance
bd = 0.6 * $rng^{2/3}$ km
Burn distance correction factor
cf = 1 - $(cb/bt)^3$
Observed burn distance
obd = bd * cf
Burnout speed
v = $rng^{1/2}$ / 11.5 km/sec
Burnout acceleration
acc = 5.5 + rng / 600 G's
Burnout flight path angle
ang = 45 * (1-rng/π Re) – 2 degrees
Time to impact after burnout
Timp = (rng – bd) / {v * cos[ang + (180/π) * rng/π Re]} sec

and is given in kilometers. Figure 1 indicates the relationship of the inputs to a TBM trajectory.

In addition to the basic input parameters, some additional derived quantities are needed to get to the final results. From the input data, it is necessary to

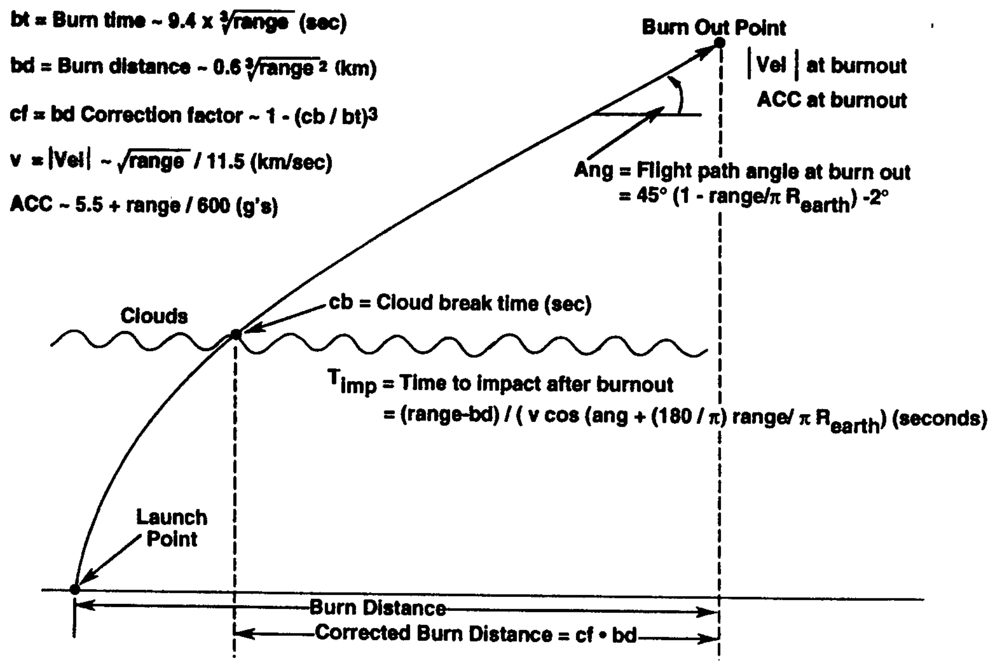


Figure 1. TBM Input Parameter Data

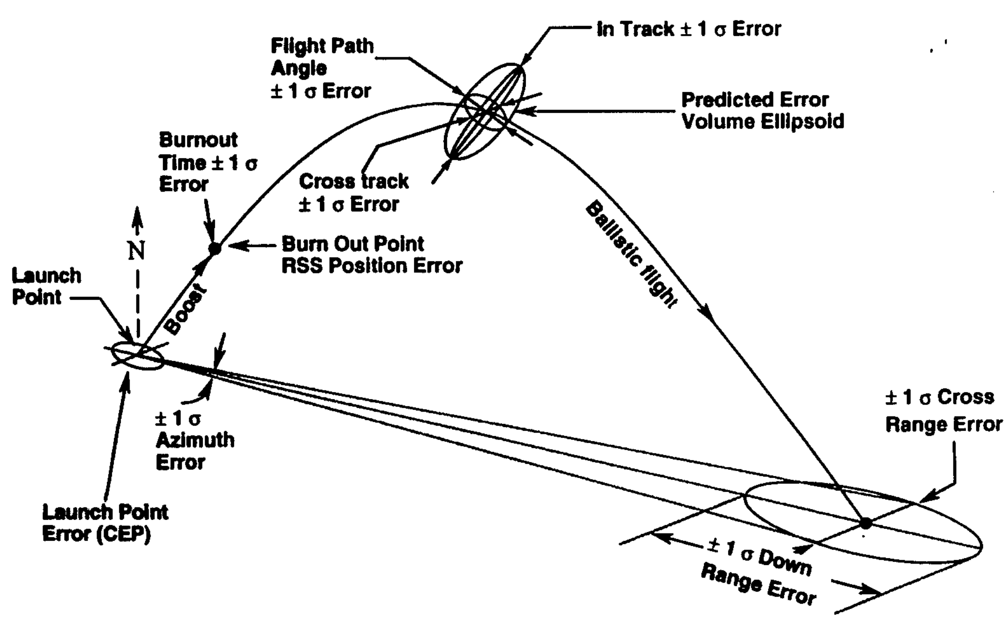


Figure 2. Error Parameter Depiction

estimate the number of stereo observation intervals that will be available to use for further calculations. This number, n, is estimated by its expected value, that is n = the integer value of [(bt – cb)/Tr – 1/2], but should be at least 1. The subtraction of 1/2 takes into account the fact that the average time of the first observation will fall at half the revisit interval after cloud break.

A correction factor, cf, is needed to account for the fact that the ballistic missile is not observable until after cloud break and the burn distance, bd, must be reduced correspondingly. This factor, which multiplies bd, was arrived at empirically and is $cf = 1 - (cb/bt)^3$. As an example, for a TBM that would fly a range of 300-km, using the nominal inputs and formulas from Table 1, bt = 62.927 seconds, cb = 30 seconds, and Tr = 10 seconds. Then n = 2 observation intervals and cf = 0.892, reducing bd from its nominal value of 26.888 km to an effective value of obd = 23.975 km for subsequent calculations (e.g.the azimuth error statistics).

OUTPUT DATA

Table 2 shows the output parameters of interest, and Figure 2 displays them graphically.

Table 2. Output Data

Position	x,y,z	km	1σ
Launch point	LP	km	CEP
Burnout point	BOP	km	RSS
Burnout time	Tbo	sec	1σ
Azimuth	az	deg	1σ
Flight path angle	fpa	deg	1σ
Cross-track velocity	Vctr	km/sec	1σ
In-track velocity	Vintr	km/sec	1σ
Cross-range impact point	crimp	km	1σ
Down-range impact point	drimp	km	1σ
Cueing area radius growth rate	cue	km/sec	1σ

CALCULATIONS

Given the inputs described in Table 1, (i.e. positions and LOS error statistics for the satellites, ballistic missile position and data on the boost trajectory until burnout, cloud break time, revisit interval, and the total number of observing satellites), all of the ballistic missile error parameters can be estimated. The first step is to estimate the statistics of the position error at the target for each observation by a stereo pair, based on the satellite error statistics and known viewing geometry, and to use these statistics to estimate the other parameters.

LOS Errors To Stereo Position Estimate Errors

Given a series of stereo position estimations, Xi, Yi, and Zi, from each satellite pair at times Ti, the assertion is that the three dimensional (3-D) stereo position errors of each observation based on the random error components of satellite LOS errors are normally distributed in X, Y, and Z (defined, respectively, as directions along and perpendicular to the true path of the ballistic missile over the ground, and the altitude above the ground), with zero means and standard deviations σx, σy, and σz. For equal satellite ranges and LOS errors, the approximate values of σx or σy are taken to be

$$\sigma x \text{ or } \sigma y = \sigma azr \text{ (or } \sigma elr) * \text{range to target} / \sin(\theta) \text{ and}$$

$$\sigma z = \sigma azr \text{ (or } \sigma elr) * \text{range to target} / 2^{1/2},$$

where θ is the bistatic angle between the viewing LOS of each satellite of a stereo pair. Since σazr = σelr by assumption, for simplicity we will use only σazr in subsequent formulas. A more detailed estimate would take into account differences in the range from each satellite and the possibility of different LOS errors for each satellite. This difference is accounted for in the "Derivations" paragraph of this paper. For the viewing geometry given above, the range to the TBM from each satellite is 37,911 km and the bistatic angle, θ, is 67.57 degrees. For σazr = 15 microradians, then σx = σy = 0.615 km or 615 m and σz = 402 m.

If more than two viewing satellites contribute observations, the position error can be expected to be somewhat less than the stereo estimate. For Nsat (the number of satellites), the expected performance can be estimated as follows: First, for each satellite stereo pair combination [k = Nsat (Nsat – 1)/2 pairs], calculate the σx, σy, and σz. Then, calculate the overall estimate for the position error standard deviation in each direction from the empirical relationship

$$(Nsat/2) / \sigma^2 = 1/\sigma 1^2 + 1/\sigma 2^2 + \ldots + 1/\sigma k^2$$

and use these quantities for additional calculations in place of the stereo values calculated earlier.

DERIVATIONS

Given information about the LOSs to the same object from two different satellites whose positions are

known, a weighted least squares estimate of the most likely position of the target object can be calculated. The initial information known about each LOS is the angular error statistics associated with it. After making an initial estimate of the object position, an estimate of the range from each satellite to the object is available. The squared product of the range and the standard deviation of the angular error for each satellite can be used in an iteration to weight the least squares estimate properly for an optimum estimate of object position.

Let these weights be $W1 = (Rng_1 * \sigma_1)^2$ and $W2 = (Rng_2 * \sigma_2)^2$ for satellites 1 and 2, respectively, and let θ be the bistatic angle between the LOS from each satellite. The bistatic angle is the angle whose cosine is the dot product of the unit vectors for each LOS. The bistatic angle is ***not*** the longitude separation between the satellites; however, for geosynchronous satellites, it is approximately the longitude difference plus 5 to 10 degrees.

The resulting position estimate will have 3-D error components along the object related x, y, and z axes of σx, σy, and σz, respectively, where x is defined, as above, as along the local horizontal direction of flight of the ballistic missile, y is perpendicular to x in the local horizontal plane, and z is in the direction of local vertical. The root-sum-square (RSS) estimate of the total error is $(\sigma x^2 + \sigma y^2 + \sigma z^2)^{1/2}$, which can be approximated by

$$W1 / \sin^2(\theta) + W2 / \sin^2(\theta) + W3,$$

$$\text{where } W3 = W1 * W2/(W1 + W2).$$

In general, the components of x, y, and z do not line up with the terms in the approximation; however, the approximation terms can be interpreted as error components along the two LOS vectors (the W1 and W2 terms) and the error component perpendicular to the plane containing those vectors (the W3 term). By equating the W3 term, which is independent of the bistatic angle, to the z axis variance (σz^2), the x and y axis terms can be equated to the remaining W1 and W2 terms. This approach to estimating the x and y errors is conservative. Since the W3 term is always the smallest term, allocating the errors of the other terms to x and y ensures that they are as large as possible.

Since the orientation of the ballistic missile flight path over the ground and its relation to the LOS vectors <u>is</u> arbitrary, there should be no preferred choice of the W1 or W2 term for either x or y. In that case, let $\sigma x^2 + \sigma y^2$ be equal to $W1 / \sin^2(\theta) + W2 / \sin^2(\theta)$, and, as before, let $\sigma x^2 = \sigma y^2$. Then,

$$\sigma x^2 = \sigma y^2 = (W1 + W2) / 2 \sin^2(\theta).$$

Further, when W1 = W2, then

$$\sigma x = \sigma y = \sigma azr * \text{range to target} / \sin(\theta), \text{ and}$$

$$\sigma z = (W/2)^{1/2} = \sigma azr * \text{range to target} / 2^{1/2}$$

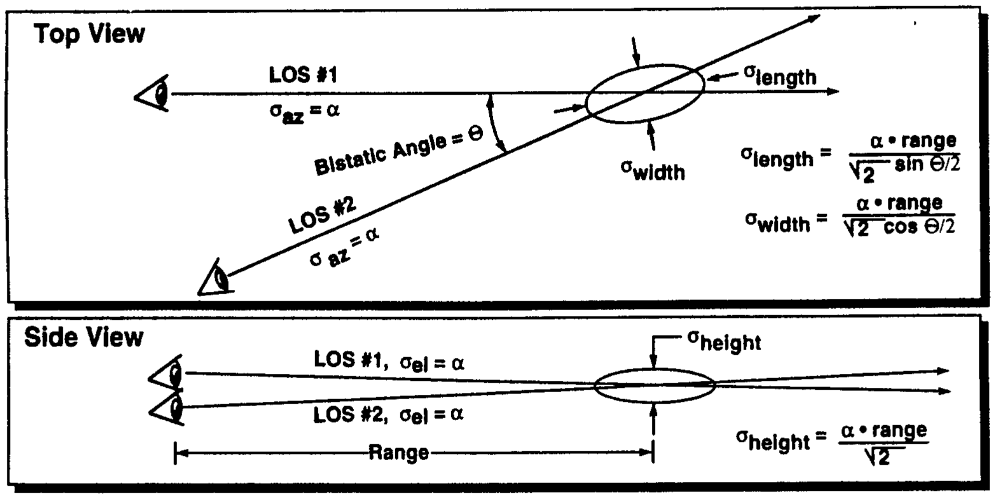
Note that the x and y position errors are minimized when the bistatic angle is 90 degrees and only grow by about 10 percent at bistatic angles of 65 or 115 degrees. Thus, satellite longitude separations over a range of about 50 degrees provide nearly equivalent performance. The z error is not dependent on the bistatic angle but only on the LOS angular error and range to target.

A different expression for the axes of the error ellipsoid resulting from the least squares estimate when W1 = W2 = W is as follows. The length direction of the error ellipsoid is in the direction of the bisector of the bistatic angle and the width direction is perpendicular to the length direction in the plane of the LOSs. The height direction is perpendicular to them both and also to the plane of the LOSs. The magnitudes of the variances of each axis error are $(W/2) / \sin^2(\theta/2)$, $(W/2) / \cos^2(\theta/2)$, and $W/2$, respectively. When the first two terms are summed, they are equal to $2W/\sin^2(\theta)$, and the same allocation to σx and σy can be made as before. Figure 3 illustrates this relationship graphically.

<u>Azimuth, and Velocity Estimates and Variances</u>

With an estimate of the position errors for each observation, we can now examine the potential errors in flight azimuth and velocity (actually speed over the ground) arising from a linear least squares fit to the n+1 errored stereo observations. We also can estimate the position errors at the beginning and end of the burn trajectory based on the same fit over all the observations made during the burn. Note that the following calculations assume simplified motions for the trajectory, which are known to be incorrect for actual ballistic missile motion and will not produce good parameter estimates. However; the assertion is that the error statistics based on these assumptions are reasonable. Figure 4 shows the projection of the x and y components of the position estimate errors to the ground.

For estimating the above errors, initially assume that for general x and y coordinate directions:



$$\sigma_x^2 + \sigma_y^2 + \sigma_z^2 = \sigma_{length}^2 + \sigma_{width}^2 + \sigma_{height}^2 = 2\left(\frac{\alpha \bullet range}{\sin\Theta}\right)^2 + \frac{(\alpha \bullet range)^2}{2}$$

Since TBM flight azimuth is arbitary; let $\sigma_x = \sigma_y = \frac{\alpha \bullet range}{\sin\Theta}$, $\sigma_z = \frac{\alpha \bullet range}{\sqrt{2}}$

Figure 3. Converting LOS Errors to Position Errors

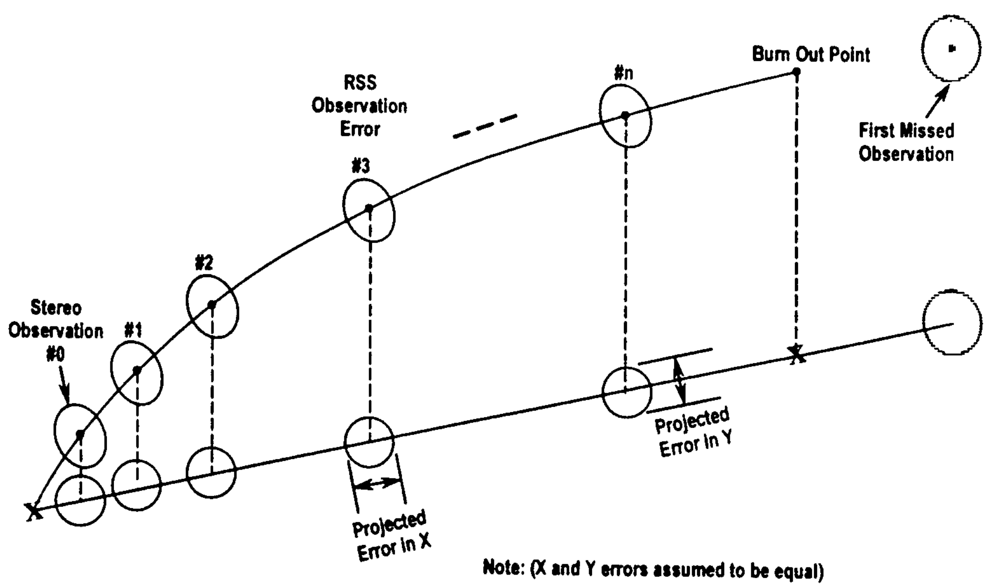


Figure 4. Stereo Position Errors to Ground Errors

$$X(t) = Xo + VX*t \text{ and } Y(t) = Yo + VY*t,$$

where Xo and Yo are the true positions at time zero and are, by definition, zero. This simplification assumes a constant speed,

$$Vo^2 = VX^2 + VY^2,$$

and direction of flight, but is found to be quite adequate for the purpose. More detailed analyses using various models for acceleration produce somewhat more optimistic, but essentially equivalent results.

Performing a linear least squares fit to the Xi data, for {ti = i * Tr | 1 = 0 to n} and solving for Xo and VX produces the following standard results as the estimates for Xo and VX.

$$\tilde{Xo} = \frac{2}{(n+1)(n+2)} \sum_{i=0}^{n} (2n+1-3i)Xi$$

$$\tilde{VX} = \frac{6}{(n+1)(n+2)\,nTr} \sum_{i=0}^{n} (2i-n)Xi$$

These estimators are from a bivariate normal distribution with means equal to Xo and VX. The variances and covariance are given by:

$$\mathrm{var}(\tilde{Xo}) = \frac{2(2n+1)}{(n+1)(n+2)} \sigma_x^2$$

$$\mathrm{var}(\tilde{VX}) = \frac{12n}{(n+1)(n+2)} \left(\frac{\sigma x}{nTr}\right)^2$$

$$\mathrm{cov}(\tilde{Xo}, \tilde{VX}) = -\frac{6}{(n+1)(n+2)} \left(\frac{\sigma_x^2}{Tr}\right)$$

Computing the variance of X(t), we have

$$\mathrm{var}(\tilde{X}(t)) = \frac{\left(12n\left(\frac{t}{nTr} - \frac{1}{2}\right)^2 + n + 2\right)}{(n+1)(n+2)} \sigma_x^2$$

Exactly analogous results for Yo, VY, and Y(t) come from regressing Y against t.

Using the above distributions, we computed the distribution for the error in azimuth, where the azimuth is given by arctan(VY/VX). This had a rather complicated distribution without a closed form expression. However, we found that it quickly converges, for parameter ranges of interest, to a simpler form found by taking the x axis in the direction of flight.

This "theoretical orientation helps in two ways. First, it simplifies simulations to enable empirical verification of the results. Second, it makes the x coordinates large with respect to the y coordinates, all of whose values come from a normal distribution with mean zero and standard deviation σy ($=\sigma x$). Since the final error estimates will be expressed in terms of parameters that are independent of the orientation, there is no loss of generality.

Using the form X = Vot +Xo, we find that that the velocity estimate and its associated error are

$$\tilde{Vo} = \frac{6}{(n+1)(n+2)\,nTr} \sum_{i=0}^{n} (2i-n)Xi$$

$$\mathrm{var}(\tilde{Vo}) = \frac{12n}{(n+1)(n+2)} \left(\frac{\sigma x}{nTr}\right)^2, \text{ or}$$

$$\sigma\, Vo = \sqrt{\frac{n+2}{(n+1)(n+2)}} \left(\frac{\sigma x}{nTr}\right),$$

where nTr is the observed time of flight. For the 300 km TBM example data, σ Vo = 43.5 m/s velocity standard error due solely to observation error.

Theoretically, the Xi can now be "corrected" to X(ti), which gives a ΔX = VoTr. The form Y = mX + Yo can then be used to regress the Yi against the X(ti). Since the Xi are, in general, very large compared to the correction, in simulations the estimates for the parameter M are very close and appear to be statistically equal.

In an equivalent form of the error estimate for Vo, the error estimate for the slope m is

$$\mathrm{var}(\tilde{m}) = \frac{12n}{(n+1)(n+2)} \left(\frac{\sigma y}{n\Delta X}\right)^2, \text{ or}$$

$$\sigma\, m = \sqrt{\frac{12n}{(n+1)(n+2)}} \left(\frac{\sigma y}{obd}\right),$$

where obd = the total observed distance flown = $n\Delta X$.

It is worth observing that, for a given position error, the velocity error is inversely proportional to the observing time and the slope error is inversely proportional to the distance flown, and both errors further decrease as the square root of the number of observations.

Since we have assumed the x axis in the direction of flight, the expected value of the error in the slope estimate is zero. The standard deviation of the azimuth estimate error, σaz, can be approximated for

small σm by the angle whose tangent is equal to σm, so that $\sigma az \sim \arctan(\sigma m)$. That is

$$\sigma \tilde{az} \approx \arctan\left(\sqrt{\frac{12n}{(n+1)(n+2)}}\frac{\sigma y}{obd}\right).$$

In an analogous way, the errors in altitude and flight path angle, can be estimated as

$$\mathrm{var}(Z(\tilde{X})) = \frac{\left(12n\left(\frac{X}{n\Delta X}-\frac{1}{2}\right)^2+n+2\right)}{(n+1)(n+2)}\sigma_z^2$$

$$\sigma \tilde{fpa} \approx \arctan\left(\sqrt{\frac{12n}{(n+1)(n+2)}}\frac{\sigma z}{obd}\right)$$

Continuing to use the earlier data for the 300 km TBM, $\sigma m = 0.0363$ or $\sigma az = 2.078$ degrees, and $\sigma fpa = 1.358$ degrees. The proper way to compute the probability that the azimuth error will be less than 5 degrees is to convert the 5 degrees to a slope by taking the tangent (= .0875) and converting that to a normalized z value by dividing by σm (= 2.41 standard deviations) and then using the cumulative normal distribution to compute the two sided probability (= 0.984). In fact, for σaz and the error bound each less than 5 degrees, assuming that the azimuth error is normal would result in a probability estimate difference of less than 0.001.

Launch/ Burnout Point Random and Bias Errors

The standard deviations of X(t), Y(X), and Z(X) are repeated here to address the launch and burnout point estimation.

$$\sigma X(t) = \sqrt{\frac{12n\left(\frac{t}{nTr}-\frac{1}{2}\right)^2+n+2}{(n+1)(n+2)}}\sigma x$$

$$\sigma Y(X) = \sqrt{\frac{12n\left(\frac{X}{obd}-\frac{1}{2}\right)^2+n+2}{(n+1)(n+2)}}\sigma y$$

$$\sigma Z(X) = \sqrt{\frac{12n\left(\frac{X}{obd}-\frac{1}{2}\right)^2+n+2}{(n+1)(n+2)}}\sigma z$$

To estimate the error in the launch point, it is reasonable to assume that launch took place at t = 0 and X = 0, the time and place of the first stereo observation. It is possible to extrapolate the launch point error estimate back to the actual position to compensate for not observing the ballistic missile until after cloudbreak; however, the improvement in the error estimate is not worth the calculation effort.

The launch point errors in x and y are statistically equal since $\sigma x = \sigma y$. We will consider the launch point estimate to be bivariate normal with zero covariance and

$$\sigma XL = \sigma YL = \sigma X(0) = \sigma Y(0) = \sqrt{\frac{2(2n+1)}{(n+1)(n+2)}}\sigma x$$

From the 300-km TBM example, σXL and σYL would be 562 meters each. These are, however, only the random part of the total launch point estimation error, and will be referred to as σ_{random}.

The errors in position caused by the bias components in the satellite LOS errors also must be considered. They will be constant for each observation, and the standard deviations in X and Y caused by bias will be equal to $\sigma_{bias} = \sigma azb$ * range to target / $\sin(\theta)$. For Z, $\sigma_{biasz} = \sigma azb$ * range to target / $2^{1/2}$. For the 300-km TBM example, $\sigma_{bias} = 2.051$ km and $\sigma_{biasz} = 1.340$ km. The estimates and statistics for Vo, az, and fpa, which are derivatives of position, will not be affected by the constant bias errors; however, the estimates for the XL and YL position errors will be affected.

When calculating the total variance for the position errors, the variances of the random components of the launch point estimate errors must be added to the variances of the bias components of those errors, producing $\sigma_{total}^2 = \sigma_{random}^2 + \sigma_{bias}^2$ in both the X and Y direction. From the 300-km TBM example, $\sigma_{total} =$ 2.126 km launch point error in X or Y.

Combining these errors in X and Y will provide an estimate of the total error of the launch point. The total launch point error, LP, is the radial distance of the estimated launch point from the true launch point, thus $\sigma LP^2 = \sigma_{totalx}^2 + \sigma_{totaly}^2$. This is a bivariate normal distribution, and, expressed in percentile form (i.e., the error corresponding to a given probability that the true target launch point error will be no greater) is:

LP = radial distance = $[-2 \ln(1\text{-probability})]^{1/2} \sigma_{total}$.

This expression indicates that the Circular Error Probable (CEP), (i.e., probability = 0.5) for a launch point estimate will be equal to 1.1774 times σ_{total}. For the 300-km TBM example, the CEP for the launch point estimate would be 2.503 km.

The launch point and burnout point error components are numerically equal, arising as opposite ends of the least squares line fit to the position data. This equality can be seen by setting $X = n\Delta X$ = obd or $t = nTr$ in the expressions for $\sigma X(t)$, $\sigma Y(X)$, and $\sigma Z(X)$. The burnout point 3-D position estimate (BOP), which would correspond to the last observation time, if no error were present in the burnout time estimate, will have a 3-D BOP RSS error equal to

$$BOP^2 = \sigma_{totalx}{}^2 + \sigma_{totaly}{}^2 + \sigma_{totalz}{}^2,$$

where $\sigma_{totalz}{}^2 = \sigma Z(0)^2 + \sigma_{biasz}{}^2$. Both the launch point and burnout point errors can be dominated by the satellite LOS bias errors , which is the case for the 300-km TBM example: the 3-D BOP RSS error is 3.313 km.

BURNOUT TIME ERRORS

Errors in estimating the burnout time can lead to an extremely large error of the burnout velocity because the ballistic missile is usually accelerating at several G's in the moments approaching burnout. The velocity errors solely caused by burnout time estimate errors are approximately equal to $G\,\Delta t$, where G is the ballistic missile acceleration and Δt is the magnitude of the burnout time error. For example, a 5-second error for a 10 G ballistic missile would lead to a velocity error of about 5s * 10 Gs * 9.8 m/s^2 = 490 m/s. After flying for 100 seconds, this would translate to a position error of 49 kilometers, possibly dwarfing all other sources of prediction error.

The assumption is that each satellite independently observes the ballistic missile at intervals of Tr seconds and that the initial observation time for each satellite is uniformly randomly distributed. Thus, the observations of two or more satellites will be asynchronous with respect to each other and also with respect to the actual burnout time, Tbo. Using the strategy of estimating Tbo as the midpoint of the time between the last observation and the first observation opportunity that does not observe the ballistic missile gives an estimate for Tbo with zero mean error and a standard deviation = σTbo

$= Tr / [2(Nsat+1)(Nsat+2)]^{1/2}$, where Nsat is the number of observing satellites. For Tr = 10, and Nsat = 1, 2, 3, and 4, σTbo = 2.89, 2.04, 1.58, and 1.29 seconds, respectively.

For Nsat satellites, the probability that the error in the estimate for Tbo will be no larger than Δt seconds is equal to

$$P(\Delta t) = [1 - (1 - 2\,\Delta t/Tr)^{Nsat}], \text{ for } \Delta t < Tr/2.$$

For example, for Tr = 10, Nsat = 2, and Δt = 3 seconds, $P(\Delta t) = P(3) = 0.84$. For a 300-km TBM accelerating at 6 G's at burnout, this 3-second error would translate to an in-track velocity error of less than 176.4 m/s 84 percent of the time. Approaching it the other way, for a given probability, the error will be less than

$$\Delta t = Tr\{1 - [1 - P(\Delta t)]^{1/Nsat}\}/2.$$

For Tr = 10 and Nsat = 2, the error will be less than 1.464 seconds 50 percent of the time.

Note that this error is only dependent on the revisit rate and the number of viewing satellites. This error is not normally distributed, but is uniform, triangular, parabolic, and so forth depending on Nsat. For Nsat >1 it can be treated as "nearly normal" to combine it with the velocity error arising solely from the observations, Vo, which is normal. Combining these errors gives a total standard deviation of the in-track velocity error, Vintr, computed from

$$\sigma Vintr^2 = \sigma Vo^2 / \cos^2(ang) + (acc * \sigma Tbo)^2 \text{ and,}$$

for the 300-km TBM example σVintr = 134 m/s.

REMAINING PARAMETERS

The remaining ballistic missile parameter errors can now be estimated. First, the cross-range impact point error, crimp, is calculated as

$$\sigma crimp = \sigma m * range \text{ km}.$$

The cross-track velocity error, Vcrtr, is then

$$\sigma Vcrtr = \sigma crimp / Timp \text{ km/sec}.$$

The down-range impact point error, drimp, can now be calculated, assuming that the orientation of the growing error ellipse stays fixed in inertial space until impact. Assuming a reentry angle equal to the burn out angle, propagating the in track velocity error for the time to impact, and accounting for the change in the local horizontal orientation after flying the ballistic missile range; the standard deviation for the down range impact point error is approximately

$$\sigma drimp = \sigma Vintr * Timp * [\sin(ang+range/Re)/\sin(ang) + \cos(2 * ang + range/Re)] \text{ km}.$$

The final quantity to be calculated is an indicator of the uncertainty area size as a function of time after burn out. This parameter, cue, is the equivalent cueing area radius growth rate and is calculated as

$$\sigma cue = (\sigma Vcrtr * \sigma Vintr)^{1/2} \text{ km/sec}.$$

This quantity is of interest in calculating the estimated area to be searched by a sensor system based on the projected error volume around the estimated position at some future time after burnout. The largest components of the error volume are the in-track and cross-track errors due to burnout position errors and the errors in estimating velocity in those directions. These components are used to bound the projected area of the 3-D volume to be searched. Cue represents the growth rate of the radius of a circle that has the same area as the projected error ellipse with major and minor axes equal to the in-track and cross-track errors. The standard deviation of the actual area to be searched will be equal to the area of a circle with a radius of the BOP RSS error plus σcue * the time after burnout. Cue is related to the bivariate normal distribution and the percentile estimate for it is approximated by

$$cue(\%) = \sigma cue * [-2 * \ln(1 - percentile)]^{1/2}.$$

For the 300-km TBM example, σcrimp, σVcrtr, σdrimp, and σcue are 10.882 km, 0.044 km/sec, 36.461 km, and .077 km/sec, respectively.

Continuing, a 95th percentile estimate of the growth rate of the radius of the search area is 188 m/s. At 100 seconds after burnout, the equivalent radius of the area to be searched for 95% confidence would be equal to BOP RSS + 100 *.188 = 3.313 + 18.8 = 22.1 km. This radius is equivalent to an area of 1535 km^2. This information can then be used to calculate the number of radar beams required to search this area for a given radar position and known capability.

SUMMARY AND CONCLUSIONS

Tables 3 and 4 summarize the completed example for the 300 km range TBM.

The expected performance of a constellation of space based IR surveillance sensors can be estimated for a wide variety of satellite constellations and viewing geometries, with error statistics for many of the tactical parameters of interest to a ballistic missile defense architecture being provided. These statistics can, in turn, be used to estimate the performance and search requirements of the ballistic missile defense system sensors. Formulas have been given to allow the calculation of all indicated parameters. An example has been given for a notional 300-km range TBM being viewed from geosynchronous altitude by two satellites spaced 60 degrees apart on each side of the TBM, with random and bias LOS errors of 15 and 50 microradians in each axis. An example using a longer range TBM/ICBM trajectory will be given in the presentation.

Table 3 Calculated Input Data for 300 km TBM

bt = 62.927 sec, bd = 26.888 km, cb = 30 sec, n = 2,
cf = .892, obd = 23.975 km, v = 1.506 km/sec,
acc = 6 Gs, ang = 42.326 degrees, Timp = 248.689 sec,
range to target = 37911 km, θ = 67.57 degrees

Table 4 Calculated Outputs for 300 km Range TBM

σx = σy = σazr * range to target / sin(θ) = 615 m
σz = σazr * range to target / $2^{1/2}$ = 402 m
σaz = atan{$[12n/(n+1)(n+2)]^{1/2}$ (σy/obd)} = 2.078 deg
σfpa = atan{$[12n/(n+1)(n+2)]^{1/2}$ (σz/obd)} = 1.359 deg
σVo = $[12n/(n+1)(n+2)]^{1/2}$ (σx/nTr) = 43.5 m/sec
σX(0) = σY(0) = σ_{random}
= $[2(2n+1)/(n+1)(n+2)]^{1/2}$ σx = 562 m
σ_{bias} = σazb * range to target / sin(θ) = 2051 m
$\sigma_{totalx} = \sigma_{totaly} = (\sigma_{random}^2 + \sigma_{bias}^2)^{1/2}$ = 2126 m
σLP = $(\sigma_{totalx}^2 + \sigma_{totaly}^2)^{1/2}$ = 3007 m
CEP = $[-2 \ln (1 - 0.5)]^{1/2} \sigma_{total}$ = 2503 m
$\sigma_{randomz}$ = $[2(2n+1)/(n+1)(n+2)]^{1/2}$ σz = 367 m
σ_{biasz} = σazb * range to target / $2^{1/2}$ = 1340 m
$\sigma_{totalz} = (\sigma_{randomz}^2 + \sigma_{biasz}^2)^{1/2}$ = 1390 m
BOP RSS = $(\sigma_{totalx}^2 + \sigma_{totaly}^2 + \sigma_{totalz}^2)^{1/2}$ = 3313 m
σTbo = Tr/$[2(Nsat+1)(Nsat+2)]^{1/2}$ = 2.04 sec
σVintr = $[\sigma Vo^2 / \cos^2(ang) + (acc * \sigma Tbo)^2]^{1/2}$
= 134 m/s
σcrimp = σm * range = 10.882 km
σVctr = σcrimp / Timp = 43.8 m/sec
σdrimp = σVintr * Timp *[cos(2 * ang + range/Re)
+ sin(ang + range/Re) / sin(ang)] = 36.461 km
σcue = (σVctr * σVintr)$^{1/2}$ = 77 m/sec

FINAL OBSERVATIONS

These calculations of error estimate statistics are general and can be applied to almost any combination of satellite viewing geometries. The formulas are suitable for incorporation into a spreadsheet from

which tables and graphs can be generated to enable comparison and visualization of the performance of the same or different satellite constellations against a wide range of target ballistic missiles.

In general, the accuracy of all parameter estimates improves as the square root of the number of observations and varies linearly with LOS errors and range of the viewing satellite from the ballistic missile.

The accuracy of launch point and burnout point position estimates will tend to be dominated by the LOS bias error. Launch point estimate errors are nearly independent of the ballistic missile range, although they do improve with a longer burn time.

Azimuth error is inversely proportional to the distance between the first and last observations and will tend to be smaller for longer range ballistic missiles. Cross-range errors in the impact point prediction tend to be nearly independent of ballistic missile range because the improved azimuth accuracy for longer range ballistic missiles is offset by the longer flight time and distance to impact.

The largest contributor to the error in predicting future position will generally be the error in estimating the burn out time (which dominates the error in estimating burnout velocity) and is proportional to the revisit rate.

A one second error in the time of burnout changes the range estimate by about 5 percent for unbussed missiles. A bus contributes an additional velocity vector to the RV.

Flight path angle errors have little effect on impact point prediction for missiles programmed for maximal distance. The errors increase to about 5 to 10 percent of the range for a one degree error on a depressed or lofted trajory targeted for half the maximal distance. This amount of deviation from the maximal distance flight path is very severe and tactically unlikely.

Uncertainty search volumes grow as the cube of time, and the projected areas as the square of time, from burnout. Since the missile is also getting closer, the search requirements grow faster than the square of the time from burnout. Long range missiles could stress radar search resources if they are not detected early.

The error in impact point prediction will be largest in the direction of the ballistic missile's estimated flight path and can be hundreds of kilometers in length. Increasing the revisit rate can dramatically reduce the burnout velocity error, and will also reduce the other errors to a lesser extent by providing more observations.